INTRODUCTION TO OPTIONS

Readings: Hull, Chapters 8, 9, and 10

Part I. Options Basics

- Options Lexicon
- Options Payoffs (Payoff diagrams)
- Calls and Puts as two halves of a forward contract: the Put-Call-Forward Parity
- P&L from options positions (P&L diagrams)
- Exchange-traded and OTC options
- Option quotes
- Options trading mechanics
- Digital (Binary) options
- Introduction to exotic options

Options Lexicon and Notation

Option
Right, but not the obligation, to buy or sell a specified quantity of the underlying asset at a specified price on or before a specified date.

Underlying (*S*)
The asset, which the option buyer has the right to buy or sell. Notation: S or $S_t = S(t)$

Call (*C*)
The right to BUY a specified quantity of the specified underlying asset.

Put (*P*)
The right to SELL a specified quantity of the specified underlying asset.

Strike/Exercise Price (*K*)
The price at which the underlying asset can be bought or sold. Notation: K

Option Expiration (*T*)
The date when the option expires. Notation: T

Time to Expiration
$\tau = T - t$, where t is today's date and T is the option expiration

European Option
The buyer has the right to exercise *only* at expiration.

American Option
The buyer has the right to exercise *at any time prior to expiration*.

Bermudan Option
The buyer has the right to exercise at expiration, as well as some additional dates prior to expiration (e.g., on the last trading day of each month or quarter between the contract inception and maturity).

Physical Settlement
When option exercise results in the purchase or sale of the actual underlying asset.

Cash Settlement
Results in the transfer of cash to the option holder amounting to the difference between the strike price and market price of the underlying asset at the time of the option exercise.

Premium
The amount paid by the option buyer to the seller (writer).

Intrinsic Value at time *t*
Call: S_t - K or 0, whichever is greater.
Put: $K – S_t$ or 0, whichever is greater.

In-the-money (ITM)
Positive intrinsic value.

Out-of-the-money (OTM)
Zero intrinsic value.

At-the-money (ATM)
The exercise price and the current underlying price are the same.

Risk-Free Rate
r: risk-free rate (continuously compounded; initially we will assume that the term structure of interest rates is flat)

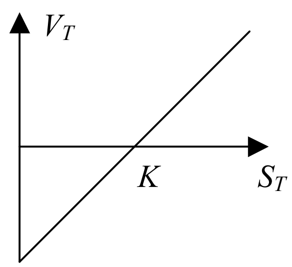
Cash Dividends
I: PV of all cash income (dividends) to be paid by the underlying asset during the option's lifetime

Dividend Yield
q: dividend yield (with continuous compounding)

Volatility
σ: volatility of the underlying asset price movements

European Calls and Puts as Two Halves of a Forward Contract: the Put-Call-Forward Parity

Consider a forward contract with delivery price K and expiration T. The payoff of the long forward position at maturity T is $V_T = S_T - K$, and the payoff diagram is a straight line:



Split the forward contract into two halves at the delivery price level K:

$$V_T = S_T - K = (S_T - K)1_{\{S_T > K\}} - (K - S_T)1_{\{S_T < K\}}$$
$$= \max(S_T - K, 0) - \max(K - S_T, 0) = (S_T - K)^+ - (K - S_T)^+,$$

where the indicator function is:

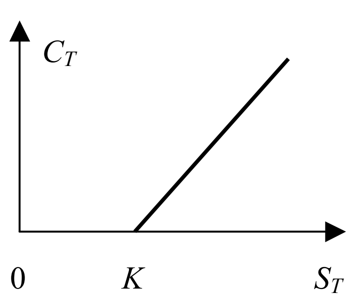
$$1_{\{S_T > K\}} = 1(0) \text{ if } S_T > K \ \ (S_T \leq K),$$

and $x^+ \equiv \max(x, 0)$ denotes the positive part of x.

Option Payoffs

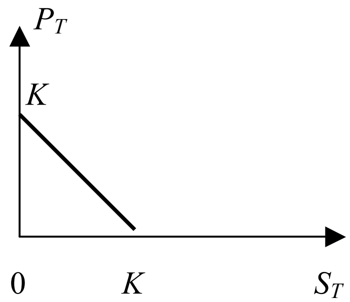
Call:

$$C_T = \max(S_T - K, 0) = (S_T - K)^+ = (S_T - K)1_{\{S_T > K\}}$$



Put:

$$P_T = \max(K - S_T, 0) = (K - S_T)^+ = (K - S_T)1_{\{S_T < K\}}$$



Put-Call-Forward Parity: the Fundamental Relationship

Let C and P be the call and put prices at time zero. Then

$$C - P = V = e^{-r\tau}(F - K),$$

where V is PV of the forward contract with the same expiration as the two options and delivery price equal to the strike price of the two options.

1. Underlying pays no dividends

$$C - P = S - e^{-r\tau}K$$

2. Known cash dividends

$$C - P = S - I - e^{-r\tau}K$$

3. Known dividend yield

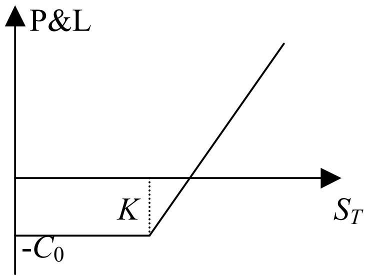
$$C - P = e^{-q\tau}S - e^{-r\tau}K$$

P&L from Options Positions

P&L Diagrams

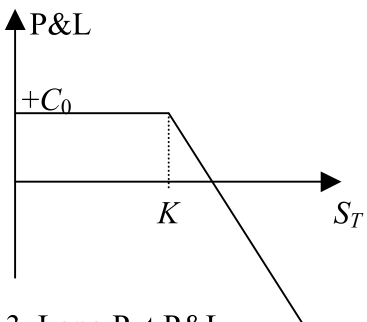
1. Long Call P&L

$+ C_T - C_0$



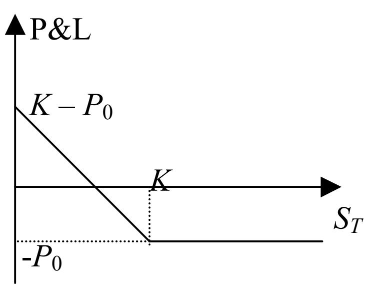
2. Short Call P&L

$- C_T + C_0$



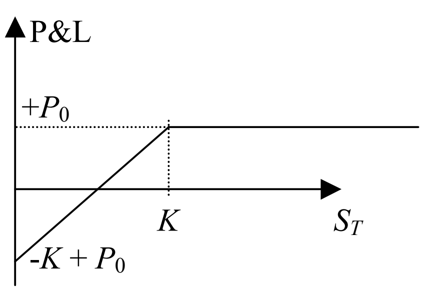
3. Long Put P&L

$+ P_T - P_0$



4. Short Put P&L

$- P_T + P_0$



Exchange-traded options

- Stock options
- Stock index options
- Currency options
- Futures options

OTC Options

Thousands of types of different option contracts: complete customization to meet specific needs.

Embedded Options (embedded in other securities, such as bonds, futures contracts)

- Convertible features
- Callable features
- Delivery options (quality and timing options)

Digital (Binary) Options

The simplest type of option is a binary bet on the direction of the underlying. This is a fundamental type of option and is a building block of more complex derivative structures.

1. Cash-or-Nothing Options

Cash-or-nothing call: $A1_{\{S_T>K\}}$

Cash-or-nothing put: $A1_{\{K>S_T\}}$

where A is a fixed (constant) dollar amount (e.g., one dollar).

2. Asset-or-Nothing Options

Asset-or-nothing call: $S_T 1_{\{S_T>K\}}$

Asset-or-nothing put: $S_T 1_{\{K>S_T\}}$

Standard call = (asset-or-nothing call) – (cash-or-nothing call)

Part II. General Properties of Option Prices
Readings: Hull, Chapter 9

- Dependence of option prices on parameters
- Simple arbitrage relationships for European options that do not require any assumptions about the underlying price process (distribution of future prices)
- Lower and upper bounds for option prices

- Early exercise of American options
- American calls
- American puts
- Effects of dividends

Effect of Variables on Option Prices

Effect of increases in variables on option prices:

Variable	C	P	C_A	P_A
S	+	-	+	-
K	-	+	-	+
σ	+	+	+	+
r	+	-	+	-
I, q	-	+	-	+
τ	+?	+?	+	+

Simple Arbitrage Relationships for Options:

Relationships for option prices that can be established by using only arbitrage arguments without any assumptions about the underlying price process.

Lower and Upper Bounds for European Call Prices:

1. Underlying pays no dividends

$$\max(S - e^{-r\tau}K, 0) \le C \le S$$

If this relationship is violated → arbitrage opportunity

First, from the call-put-forward parity we have:

$C = V + P$.

Since $P \ge 0$, we have

$S - e^{-r\tau}K \le C$.

The call option has non-negative payoff, so its price (present value) is also non-negative.

On the other hand, the call option gives the right to buy the underlying asset → its price can never be greater than the price of the underlying asset itself, so

$C \le S$.

2. Dividends

Same argument applies for the cases with known cash dividends or dividend yield because the put-call-forward parity still holds (just use the appropriate expression for the present value of the forward contract *V*).

European Calls: Is there an arbitrage opportunity?

C = \$3
S = \$52
τ = 1 year
K = \$50
r = 5%
I = 0

$S - e^{-rT} K = 52 - e^{-0.05x1}\ 50 = 4.44$

\$3 < \$4.44 → the call is *underpriced* → arbitrage opportunity to make \$1.44!

Lower and Upper Bounds for European Put Prices

$$\max(e^{-r\tau}K - S, 0) \leq P \leq e^{-r\tau}K$$

If put prices violate these inequalities → arbitrage opportunities.

First, the put-call-forward parity is:

$P = C - V.$

Since $C \geq 0$ and $P \geq 0$, we have:

$$\max(e^{-r\tau}K - S, 0) \leq P.$$

On the other hand, the maximum payoff from the put is equal to the strike *K* (realized if the stock price goes to zero). The present value of the strike is equal to $e^{-r\tau}K$, so the value of the put should always be less than the present value of the strike.

European Puts: Is there an arbitrage opportunity?

P = \$3
S = \$48
τ = 0.25
r = 5%

K = \$50
D = 0

$e^{-r\tau}K - S = 50\ e^{-0.05x0.25} - 48 = \$1.34$

\$1.34 < \$3

No lower bound violation.

Some Further Arbitrage Properties

1. Upper bound for a call spread

$C(K_1) - C(K_2) \leq e^{-r\tau}(K_2 - K_1)$, for $K_1 < K_2$

2. Convexity

Call price is a convex function of the exercise price K, i.e.,

$$C(K_2) \leq \left(\frac{K_3 - K_2}{K_3 - K_1}\right) C(K_1) + \left(\frac{K_2 - K_1}{K_3 - K_1}\right) C(K_3),$$

for $K_1 < K_2 < K_3$

3. Portfolio property

A portfolio of options is worth at least as much as an option on portfolio.

Practical Problems with Executing Option Arbitrage Trades:
- Synchronizing option and underlying asset trades
- Transaction costs
- Dividends must be estimated

Early Exercise of American Options

Example: Options on S&P 500 (SPX) are European, and on S&P 100 (OEX) are American.

American option = European option + the right to exercise early. Hence, American options are *never* worth less than European options.

American Calls (No Dividends)

Suppose you are holding an ITM American call struck at 60, $S = 100$, $\tau = 0.25$, $I = 0$.

What should you do: hold to expiration or exercise now?

Reasons for *not* exercising a call early (no dividends):

- No income from the stock is sacrificed
- Strike price is paid later (buy the asset at the strike) – time value of money advantage tells you to hold the option rather than exercise early
- Insurance premium of the option retained

If you do not want to hold the call any longer, just sell it and receive its market value, but do not exercise.

1. Suppose you do exercise at some time t^* before maturity. You get the stock worth $S(t^*)$ and pay the strike price K (need to borrow K dollars to pay for the stock). At time t^*: $S(t^*) - K$.

At time T your position is worth:
$S_T - e^{r(T-t^*)}K$.

2. Alternatively, suppose you decide to hold to expiration. Then at time T your payoff is:
$\max(S_T - K, 0)$.

Since $\max(S_T - K, 0) > S_T - e^{r(T-t^*)}K$ irrespective of the outcome for the random variable S_T, i.e., the call option payoff dominates the payoff of the alternative strategy of exercising at t^*, you should never exercise a call early.

Price of American Call = Price of European Call

If you do exercise a call early, you lose:
- Interest on the amount K you will have to pay until maturity T
- Insurance premium of the ITM call

American Puts (No Dividends)

Should Puts be exercised early?

Are there any advantages to exercising an ITM put early? Consider first the case with no dividends.

American Put:
Even in the absence of dividends it may be optimal to exercise an ITM put before maturity. This happens when the stock falls low enough that the benefit received from the likelihood that it falls even more is less than the interest that can be earned on the cash received from immediate exercise.

Suppose we exercise an ITM put at time t and receive the strike price K for the stock worth S. We can invest K at r and carry our short position in the stock through expiration T. The value of our portfolio at T is then:

$$e^{r(T-t^*)}K - S_T . \qquad (*)$$

On the other hand, if we hold on to the option, we get the payoff:

$$\max(K - S_T, 0) . \qquad (**)$$

For outcomes with $S_T < e^{r(T-t^*)}K$, (*) is greater. For outcomes with $S_T > e^{r(T-t^*)}K$, (**) is greater.

- Time value of money tells us we should exercise and invest the sale proceeds to earn the interest
- But if we exercise we sacrifice the insurance value left in the put

Thus we exercise only if interest on the strike K is greater than the remaining insurance value.

American put: $P_A = P + E$,
where
P: European put
E: value of the right to exercise early. Unfortunately there is no simple analytical formula for the value of this option to exercise early even in the Black-Scholes-Merton model.

The arbitrage lower bound for American puts:

$\max(K - S, 0) \le P_A$
Compare with the lower bound for European puts:
$\max(e^{-r\tau}K - S, 0) \le P$.

Effects of Dividends on American Option Prices

American calls and dividends
Dividends reduce the value of a call option (since they depress the ex-dividend stock price).

<u>1. Cash dividends</u>
Sometimes it is optimal to exercise an American call *just prior to an ex-dividend date.* This is because the dividend will cause the stock price to jump down. It is never optimal to exercise at other times.

<u>2. Constant Dividend Yield q</u>
Suppose we exercise at time t and pay K for the asset worth S. Then at time T we have (we collect dividends from the asset and reinvest them back in the asset):

$$e^{q(T-t)}S_T - e^{r(T-t)}K. \qquad (*)$$

If we hold we get the payoff:

$$\max(S_T - K, 0) \qquad (**)$$

By holding the option rather than the asset, we are sacrificing dividends (option does not pay dividends). But if we exercise we sacrifice the insurance value.

There is no closed-form analytical formula for the price of American calls with $q > 0$.

American puts and dividends

Dividends increase the insurance premium value of the put (since they depress the underlying price). Other things being equal, there is a high enough dividend when it becomes optimal to hold the ITM put rather than exercise it (assuming it is optimal to exercise an identical put in the situation with no dividends).

Early Exercise Summary:
- Calls (no dividends): never optimal to exercise early
- Calls (cash dividends): may be optimal to exercise ITM calls just prior to an ex-dividend date.
- Calls (dividend yield): may be optimal to exercise ITM calls early
- Put (no dividends): maybe optimal to exercise ITM puts
- Puts (dividends): dividends increase the value of the put. If dividends are high enough, it may not be optimal to exercise in a situation where it would be optimal to exercise without dividends.

Part III. Options Positions and Trading Strategies
Readings: Hull, Chapter 10

Applications of Option Strategies:

1. Insurance
Options are purchased to protect against adverse changes in the value of underlying asset, while retaining profit opportunity.

2. Income (Yield) Enhancement
Options are sold to provide income in order to increase returns or lower costs.

3. Trading/Speculative Positions
Options are purchased and sold to take positions to profit on expected market or volatility moves.

4. Trading/Arbitrage Positions
To capitalize on mispricing of options vs. the underlying asset or options vs. options.

Types of Trading Strategies with Options

- An option plus a position in the underlying
- Two or more options of the same type (calls or puts): Spreads
- A mixture of calls and puts: Combinations

General Option Strategy:

$$Strategy = \sum_i \alpha_i C_i + \sum_i \beta_i P_i + \gamma S$$

The variations are virtually unlimited!

Analyzing Option strategies:

- Understand the payoff structure and payoff diagrams (plot a payoff diagram for each component and then net them together to arrive at the payoff from the net position)
- Does the strategy result in debit or credit initially?
- Construct a P&L Diagram
- What market view is appropriate for the position? (up, down, stable, volatile, other)
- What is profit/loss potential? (unlimited/limited)
- Is the primary purpose insurance, yield enhancement, or trading profits (speculation or arbitrage)?

Summary of Option/Underlying Strategies

1. Covered write (call) $+ S - C$
Purpose: Yield Enhancement
Market View: Moderately bullish, volatility stable or declining
Profit: Limited Loss: Open

2. Long call to protect short position: $- S + C$
Purpose: Insurance
Market View: Moderately Bearish, volatility increasing
Profit: Open Loss: Limited

3. Protective put strategy: $+ S + P$
Purpose: Insurance
Market View: Moderately Bullish, volatility increasing
Profit: Open Loss: Limited

4. Covered write (put): $- S - P$
Purpose: Yield Enhancement
Market View: Moderately Bearish, volatility stable or decreasing
Profit: Limited Loss: Open

One can also sell a put as a strategy to acquire the stock.

Some important spreads:
1. Bull call spread (vertical call spread)
2. Bear put spread (vertical put spread)
3. Butterfly call spread
4. Butterfly put spread

Some important combinations:
1. Synthetic long forward
2. Synthetic short forward
3. Straddle
4. Strangle

Some well-publicized losses on option strategies

- October 1987 First Options Corp: stock options customers with program of writing uncovered puts. One made \$20 million over previous four years, gave it all back plus \$60 million in one day. First Options went out of business.

- Barings Bank: Short naked straddles on the Japanese stock index Nikkei 225. When the straddles got skewed after the market went down, Nick Lesson doubled up!

In addition to plain vanilla call and put options discussed in this notes, there are a large variety of different types of options traded over-the-counter. By the nature of their payoffs, option contracts can be classified as follows:

1. Single variable path-independent

Payoff depends on the value of a single underlying variable at one specific date in the future (often, contract expiration) (e.g., European options)

2. Single-variable path-dependent

Payoff depends on the history of the underlying variable from the contract inception to contract expiration - *path-dependent options* (e.g., American options, barrier, Asian, lookback options)

3. Multi-variable path-independent

Payoff depends on the values of several underlying variables at one date in the future (e.g., European-style basket options, out-performance options)

4. Multi-variable path-dependent

Payoff depends on the history of several underlying variables (e.g., Asian-style basket options)